MMM Math

1. ,

Where R is revenue, S is media spend, k is the reg constant, I is an index over variables. is the regression parameter indexed by i. 0 < C < M, are hyperparameters.

For MMM we can use (1) and fit the parameters, , in log space, using linear techniques.

1a) ,

1b) ,

To understand (1), we take the derivative and then calculate the elasticity.

Derivative done by hand and by derivative-calculator.net

1. .

Elasticity is then,

1. ,

Rearranging (2) gives,

1. .

In (3), the factors are:

* 1. , the fit parameter
  2. A constant
  3. .

This shows that the elasticity is 0, at both 0 and inf, and, has the sign of gamma in between.

We find the inflection point for the elasticity by taking the derivative of (3) and setting it to 0. This is where the elasticity starts to fall off.

1. .
2. .
3. Some definitions:
   1. This is the average value of the media channel when it is used. Loosely, it is some central value of S determine either by data or as an input. We’ll scale S, M and C by this number for each channel.
   2. ,
   3. (lower case “c”),
   4. (lower case “m”).

We can now rewrite (1) (and dropping subscripts) as,

1. .

Showing the marketing response function (1) is unchanged with the transformed variables.

Substituting into (2) we get the partial derivatives,

1. .

For optimization we want the partial derivatives relative to spend, not scaled spend. To get this we can use the chain rule,

1. ,
2. \* .

At equilibrium,

1. ,

Which says that the marginal return for any two media variables, i and j, should be equal at their level spend, when .

By definition 5b,

1. \* \* .

R, revenue, cancels. If we assume, then c and m also cancel and (13) becomes,

1. .

Equation (14) says that the coefficients of the scaled variables in proportion to the average spend implies equal marginal return.

Next we’ll tweak the response function so the coefficients themselves are directly comparable. That is, equal coefficients imply equal marginal return at spend level S\_bar.

1. ,

Equation (13) is then,

13a)

Substituting back into (6),

1. ,

In log space,